Chaozheng Li et al.: Processing DOR Signal using Chirp-z Transform Correlator, IVS 2012 General Meeting Proceedings, p.301–305
http://ivscc.gsfc.nasa.gov/publications/gm2012/li-chaozheng.pdf

Processing DOR Signal using Chirp-z Transform Correlator

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Abstract

VLBI is a powerful tool for spacecraft tracking. The spacecraft-based Differential One-way Ranging (DOR) beacon emits a series of tones other than the continuous spectrum signal. Chirp-z transform (CZT) is a kind of zoom spectrum analysis method, and it is suitable for DOR signals. A VLBI correlator which has the CZT algorithm and which is appropriate for DOR signal analysis is under development. Theoretical analysis and experiment results indicate that CZT is able to obtain higher spectral resolution and is faster than the regular Fast Fourier Transform (FFT) method.

1. Background

The normal FX-type Very Long Baseline Interferometry (VLBI) correlator uses a Fast Fourier Transform (FFT) to analyze coherence spectra. But there are some restrictions of FFT: the FFT frequency resolution is restricted by f_s/N. Here, f_s is the sample frequency and N is the number of FFT points. The number of FFT operations is closely related to N. Currently China is carrying out Lunar and Martian explorations. The special spacecraft-based Differential One-way Ranging (DOR) beacon used for VLBI observation is made up of a series of tones other than the continuous spectrum signal. To get precise VLBI results, it is necessary to study the new DOR processing method of the VLBI correlator.

2. Chirp-z Transform

The CZT is a kind of zoom spectrum analysis method. It can recognize the fine spectrum structure and resolves it in more detail [1]. The CZT samples in the frequency domain along an arbitrary spire in the Z-plane. An M-element CZT can be defined as

$$X(e^{j\omega_k}) = W^{k^2/2}(\sum_{n=0}^{N-1} g[n]W^{-(k-n)^2/2}), k = 0, 1, \ldots, M-1 \tag{1}$$

where ω_0 is the initial phase, $\Delta\omega$ is the phase step, $W = \omega_0 \cdot e^{j\Delta\omega}$ and $g[n] = x[n]e^{-j\omega_0 n}W^{n^2/2}$.

The CZT results, $X(e^{j\omega_k})(0 \le k \le M-1)$, are the equivalent interval samplings at frequency $\omega_k = \omega_0 + k\Delta\omega(k = 0, 1, \ldots, M-1)$ of the continuous spectrum of $x(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{j\omega n}$. When $\Delta\omega < 2\pi/N$, the CZT frequency sampling interval is less than the FFT, so CZT zooms the spectrum interval of $[\omega_0, \omega_0 + (M-1)\Delta\omega]$ and gets the spectrum details.

According to [1], it seems to have infinite spectrum resolution, but actually the spectrum resolution is restricted by the length of data and the window function type [2].

3. Comparison between FFT and CZT

3.1. Computation Precision

1. Frequency Precision Analysis

   When the frequency of the signal component coincides with the FFT sampling point, there is no frequency error. But suppose that the actual frequency of the signal component occurs in the middle of two FFT sampling points ω_i and ω_{i+1}, where $\omega_i = \omega_0 - \Delta\omega/2$ and $\omega_{i+1} = \omega_0 + \Delta\omega/2$. Here ω_0 is the non-unitary digital frequency; the frequency error could be $\pm 0.5\Delta\omega$. If the spectrum is enlarged N times by CZT, the spectrum interval is $\Delta\omega/N$. So the corresponding maximum frequency error will decrease to only $\pm 0.5\Delta\omega/N$.

2. Amplitude Precision Analysis

   According to the frequency-domain convolution theorem, the time-domain product of the window function and the single-frequency signal is equivalent to the frequency-domain convolution. Also the spectrum amplitude of the convolution is the window function's spectrum amplitude with a frequency shift. So the relative error of the window function's spectrum amplitude is the relative error of the single frequency by windowing, the frequency error interval is $[-0.5\Delta\omega, 0.5\Delta\omega]$, and the normalized frequency error interval is $[-0.5, 0.5]$. The normalized interval contains the principal window function energy and the main spectrum amplitude error, so only this interval needs selective analysis.

   The error function of rectangular window functions is $1 - |\frac{\sin(\pi\omega)}{\pi\omega}|$. The maximal relative error is 36.3%, and the maximum amplitude gain of CZT is 3.92 dB.

3. Phase Precision Analysis

   The phase of the rectangular window function is

   $$\varphi = -\omega\pi/\Delta\omega \tag{2}$$

   where $\omega \in [-0.5\Delta\omega, 0.5\Delta\omega]$.

   In the convolution operation of the spectra of the window function and the single-frequency signal, the phases of the spectra are combined. At the point of $\omega = \pm 0.5\Delta\omega$, the phase error reaches the maximum of $\pm\pi/2$.

   If zoomed N times by CZT, the interval of the spectrum lines is ω/N. Then the range of ω will be $[-0.5\Delta\omega/N, 0.5\Delta\omega/N]$. The theoretical maximum phase error does not increase with N.

3.2. Number of Operations

1. Number of CZT operations [4]

   An M-element CZT requires $S_{CZT_\times}$ real multiplication operations and S_{CZT_+} real additions:

   $$S_{CZT_\times} = 6(M+N)\log_2(M+N) + 21N + 10M \tag{3}$$
   $$S_{CZT_+} = 9(M+N)\log_2(M+N) + 6N + 4M \tag{4}$$

   where N is the number of points in the source data.

Table 1. Operations of each CZT calculation step.

Calculation Steps	Detail Steps	Real Addition	Real Multiplication
$g[n] = x[n]e^{-j\omega_0 n}W^{nk}$	$e^{-j\omega_0 n}$	0	N
	$W^{\frac{n^2}{2}}$	0	$4N$
	Multiplication	$4N$	$8N$
$\sum_{n=0}^{N-1} g[n]W^{-\frac{(k-n)^2}{2}}$	$W^{-\frac{(k-n)^2}{2}}$	0	$4N$
	Convolution	$9L\log_2 L + 2L$	$6L\log_2 L + 4L$
$W^{\frac{k^2}{2}}(\sum_{n=0}^{N-1} g[n]W^{-\frac{(k-n)^2}{2}})$	$W^{\frac{k^2}{2}}$	0	$2M$
	Multiplication	$2M$	$4M$

$L = M + N$

2. Comparison between FFT and CZT

In a radix-2 FFT operation, N points need real multiplication operations and real additions:

$$S_{FFT_\times} = 2N\log_2 N, \quad (5)$$
$$S_{FFT_+} = 3N\log_2 N. \quad (6)$$

Given N_0 FFT points and M CZT points, where $M = N_0/2$, the frequency resolution magnification factor is A, where the initial numerical frequency range is $[0, \pi]$. With the increase of A, the FFT can keep the same resolution as the CZT by zero-padding, and the corresponding FFT points N is $A \cdot N_0$. A comparison of the number of CZT and FFT operations with the same frequency resolution is shown in Figure 1.

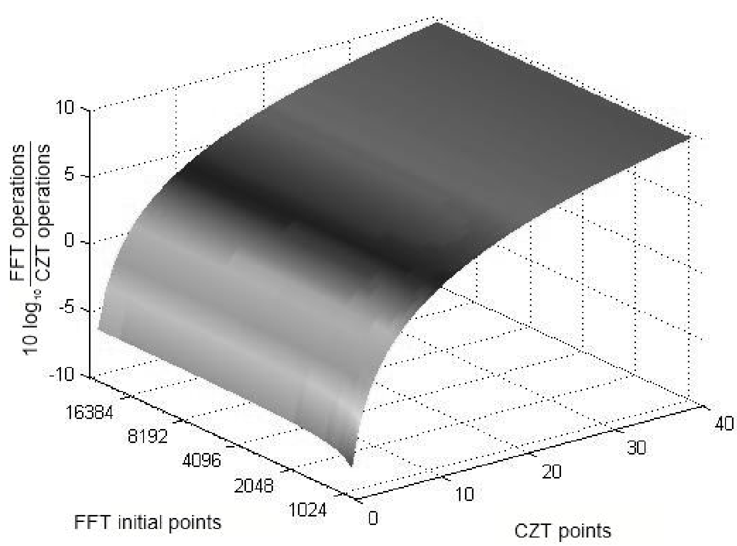

Figure 1. Comparison of the CZT and FFT operations with the same frequency resolution.

When $A = 40$, the number of FFT operations is ten times as many as the number of CZT operations. When $A < 5$, the number of FFT operations is fewer than the number of CZT operations.

4. Actual Signal Analysis

Signal 1 and signal 2 are two four-minute actual DOR signals, which are analyzed by the correlator with normal FFT (N-point) and CZT (M-point) arithmetic respectively, where M=512, N=1024, and A=32. The self-spectra results are presented in Figures 2 and 3. The cross-spectra results are presented in Figure 4 and Figure 5.

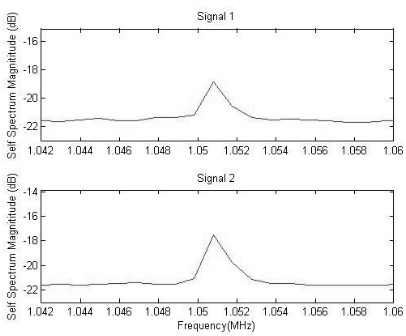

Figure 2. FFT Self Spectrum.

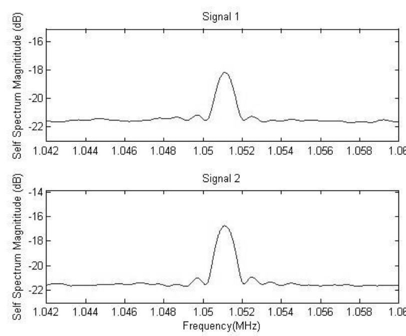

Figure 3. CZT Self Spectrum.

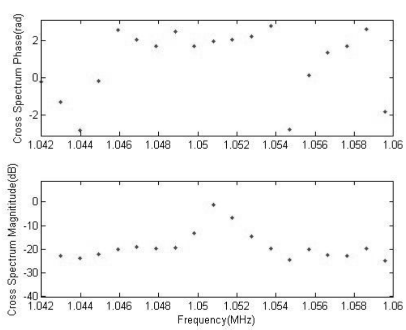

Figure 4. FFT cross-correlation.

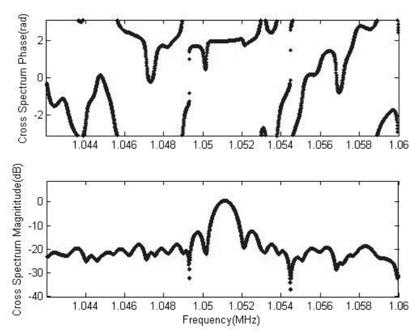

Figure 5. CZT cross-correlation.

Obviously, more details can be obtained by CZT, as well as higher frequency precision. In the single-frequency signal, the delay rate is equal to the ratio of the phase drift speed to the signal frequency. So the precision of delay rate can be improved with increased frequency precision.

A paired t-test was performed to determine if the CZT was effective in improving the Peak Signal-to-Noise Ratio (PSNR). The mean PSNR improvement (M=0.011931, SD =0.000141, N= 57) was significantly greater than zero, with t(56)=84.55436 and two-tail $p = 9.168814 \times 10^{-61}$, providing evidence that the CZT is effective in improving PSNR. A 95% Confidence Interval (C.I.) about the mean weight loss is (0.011648, 0.12214).

The time-varying phase of the CZT spectrum frequency point and that of the FFT are presented in Figure 6 and Figure 7.

After 25-order polynomial fitting, the residual delay rate calculated by the CZT is 0.5736, while the residual delay rate calculated by the FFT is 0.7921, which is 38% larger than the CZT's. So the CZT can reach higher precision than the FFT.

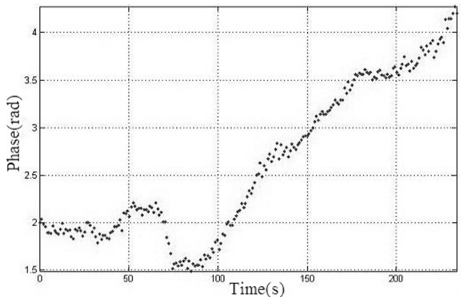

Figure 6. FFT Residual Delay Rate.

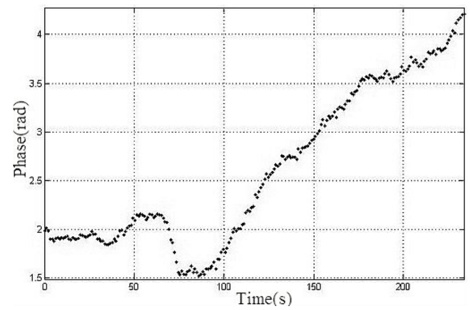

Figure 7. CZT Residual Delay Rate.

5. Summary

This paper introduces the CZT arithmetic of the VLBI correlator for DOR signal correlation. Precision analysis and operations comparison are performed. The experiment results indicate that the CZT can get higher frequency resolution and higher amplitude precision than the normal FFT, without losing phase precision. With the same spectral resolution, the number of CZT operations is fewer than that of the FFT, when the frequency resolution amplification factor is not less than 5. And in the actual data processing, the CZT can remarkably increase the PSNR and the precision of the delay rate. Because of the similarity between DOR signals and the line spectrum signal, this method can be applied to spectral line radio source data processing.

Acknowledgements

This paper is supported by the Natural Science Foundation of China General Program (11173052), the state CNGI demonstration project, the Science & Technology Commission project of Shanghai Municipality (06DZ22101) and the Chinese lunar exploration project. The authors appreciate the support from CVN Data Center and the Key Laboratory of Radio Astronomy of Chinese Academy of Sciences.

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